FORM TP 2019148

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MAY/JUNE 2019

CARIBBEAN EXAMINATIONS COUNCIL

CARIBBEAN ADVANCED PROFICIENCY EXAMINATION®

APPLIED MATHEMATICS

MATHEMATICAL APPLICATIONS

UNIT 2 – Paper 02

2 hours 30 minutes

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READ THE FOLLOWING INSTRUCTIONS CAREFULLY.

1. This paper consists of THREE sections: Discrete Mathematics, Probability and Distributions, and Particle Mechanics.
2. Answer ALL questions.
3. Write your answers in the spaces provided in this booklet.
4. Do NOT write in the margins.
5. Unless otherwise stated in the question, all numerical answers MUST be given exactly OR to three significant figures as appropriate.
6. If you need to rewrite any answer and there is not enough space to do so on the original page, you must use the extra lined page(s) provided at the back of this booklet. **Remember to draw a line through your original answer.**
7. **If you use the extra page(s), you MUST write the question number clearly in the box provided at the top of the extra page(s) and, where relevant, include the question part beside the answer.**
8. You may use a silent, non-programmable calculator.

A001

Examination Materials:

Mathematical formulae and tables **(Revised 2010)**

DO NOT TURN THIS PAGE UNTIL YOU ARE TOLD TO DO SO.

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SECTION A

MODULE 1: DISCRETE MATHEMATICS

Answer BOTH questions.

1. (a) A company manufactures two products, X and Y. Each of these products requires a certain amount of time on the assembly line and a further amount of time in the finishing shop. Product X needs 5 hours for assembly and 2 hours for finishing, and Product Y needs 3 hours for assembly and 4 hours for finishing. In any given week, the firm has available 105 hours on the assembly line and 70 hours in the finishing shop. The firm can sell all it can produce of each item and makes a profit of \$200 on each item of X and \$160 on each item of Y.

Let x represent the number of items of Product X and y represent the number of items of Product Y.

(i) On the grid provided **on page 5**, plot the feasible region for the linear programming problem which is satisfied by the following equations:

Maximize	$P = 200x + 160y$
subject to	$5x + 3y \leq 105$
	$2x + 4y \leq 70$
	$x > 0$
	$y > 0$

[8 marks]

(ii) Hence, use the linear programming problem to determine the maximum profit, P.

[4 marks]

(b) A team of 4 workers, Jairaj, Haniff, Rasleigh and Moses, are to be assigned to 4 tasks, 1, 2, 3 and 4. Each worker must be assigned to 1 task only and each task must be done by only 1 worker.

Haniff cannot be assigned to Task 4.

The amount of money, in dollars, that each worker would earn while assigned to each task is shown in the table below.

Worker	**Task**			
	1	2	3	4
Jairaj	18	24	22	17
Haniff	20	25	19	-
Rasleigh	25	24	27	22
Moses	19	26	23	14

(i) Reducing rows first, use the Hungarian algorithm to obtain an allocation that maximizes the total amount of money, in dollars, earned by the team. You must make your method clear and show the table after EACH stage.

(ii) Determine the total amount of money earned by the team based on the assignment.

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[2 marks]

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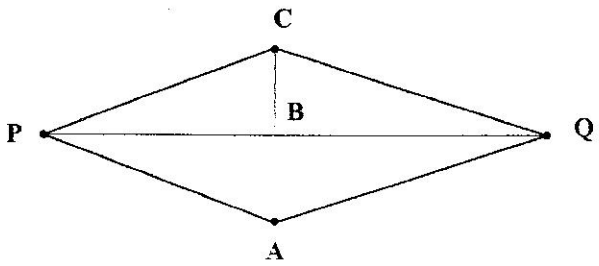
(c) Given the following diagram,



list any THREE closed paths which start at P.

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[3 marks]

Total 25 marks

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2. (a) The Boolean function for a given network is $F = (r \vee s) \wedge t$. Design a switching circuit to illustrate this function.

[4 marks]

(b) Construct a truth table to show that the proposition

$p \wedge q$ logically implies $p \leftrightarrow q$.

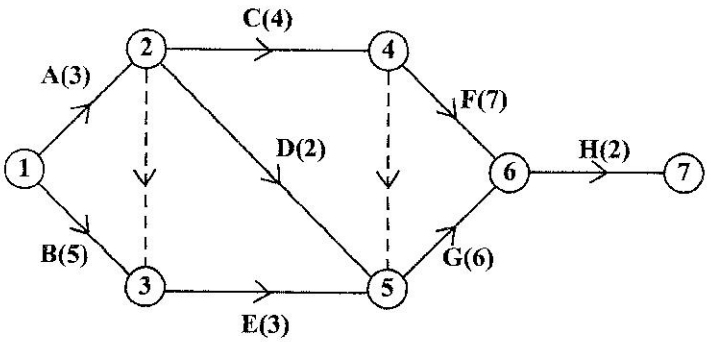
[5 marks]

(c) Draw a logic circuit corresponding to the Boolean expression

$a \wedge (\sim b \vee c)$.

[5 marks]

(d) The following diagram is an activity network relating to a building project. The number in brackets on each arc is the time taken, in days, to complete the activity.



(i) Explain the significance of the broken line from Event 2 to Event 3.

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[2 marks]

(ii) Construct a precedence table to represent the information in the activity network above.

[3 marks]

(iii) Complete the following table to show the earliest start time and the latest start time for EACH activity.

Activity	Earliest Start Time	Latest Start Time
A		
B		
C		
D		
E		
F		
G		
H		

[4 marks]

(iv) Determine the critical activities and the length of the critical path.

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[2 marks]

Total 25 marks

SECTION B

MODULE 2: PROBABILITY AND DISTRIBUTIONS

Answer BOTH questions.

3. (a) The lifetime, x, in tens of hours, of a battery has a cumulative distribution function $F(x)$ given by

$$F(x) = \begin{cases} 0, & x < 1 \\ \frac{4}{9}(x^2 + 2x - 3), & 1 \leq x \leq 1.5 \\ 1, & x > 1.5. \end{cases}$$

(i) Determine the median of x, giving your answer to 3 significant figures.

[3 marks]

(ii) Determine, in full, the probability density function of the random variable x.

[3 marks]

(iii) Calculate $P(x \geq 1.2)$.

[2 marks]

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(iv) A camping lantern operates on 4 batteries, all of which must be working. Four new batteries are put into the lantern.

Find the probability that the lantern will be working after 12 hours.

[2 marks]

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(b) A large number of regular users log into Company X's website. On average, 4 users every hour fail to connect to the company's website on their first attempt.

(i) State ONE reason why the Poission distribution may be a suitable model in this case.

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[1 mark]

(ii) Find the probability that in a randomly chosen **two-hour** period none of the users fail to connect on their first attempt.

[2 marks]

(iii) Find the probability that in a randomly chosen **two-hour** period at least 4 users fail to connect on their first attempt.

[3 marks]

(c) A committee of 6 persons must be selected from a group of 10 men and 9 women.

Determine the number of possible committees that can be chosen if the committee consists of

(i) at least 4 men and 1 woman.

[3 marks]

(ii) no men.

[3 marks]

(iii) The committee that is chosen consists of 4 men and 2 women. The members of the committee randomly queue for refreshments. Find the probability that the women are NOT next to each other.

[3 marks]

Total 25 marks

4. (a) A manufacturer knows that for a certain cloth, faults occur randomly during its production at a rate of 2 faults every 15 metres. Determine the probability of

(i) exactly 4 faults occurring in a 15 metre length of cloth

[2 marks]

(ii) more than 10 faults occurring in a 60 metre length of cloth.

[3 marks]

(iii) A retailer buys a large amount of this cloth and sells it in pieces of length y metres. He chooses y so that the probability of no faults in a piece of cloth is 0.80.

Write an equation for y and show, correct to 2 significant figures, that $y = 1.7$.

[4 marks]

(iv) A retailer sells 1200 of these pieces of cloth. He makes a profit of $6 on EACH piece of cloth that does not contain a fault and a loss of $15 on any piece that contains faults.

Calculate the retailer's expected profit.

[4 marks]

(b) A local manufacturing company requires that prospective employees be interviewed by 3 different executives. This enables the company to obtain a consensus evaluation of each candidate. Each executive gives the candidate either a positive zero or negative rating. The following table shows the interview results of the last 100 candidates.

Rating	0	1	2	3
No. of Candidates	18	47	24	11

For greater efficiency, the director of recruitment suggests that the interview process can be approximated by a binomial distribution with $p = 0.40$; that is, with a 40% chance of any candidate receiving a positive rating in any one interview.

A chi-squared test is carried out to determine whether the director's theory is justified.

(i) State the

- null hypothesis

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[1 mark]

- alternative hypothesis.

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[1 mark]

(ii) Determine the expected frequencies if the null hypothesis is true.

[4 marks]

GO ON TO THE NEXT PAGE

(iii) Using a 10% significance level, determine whether the director's theory can be accepted. Clearly state the conclusion of your test.

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[6 marks]

Total 25 marks

SECTION C

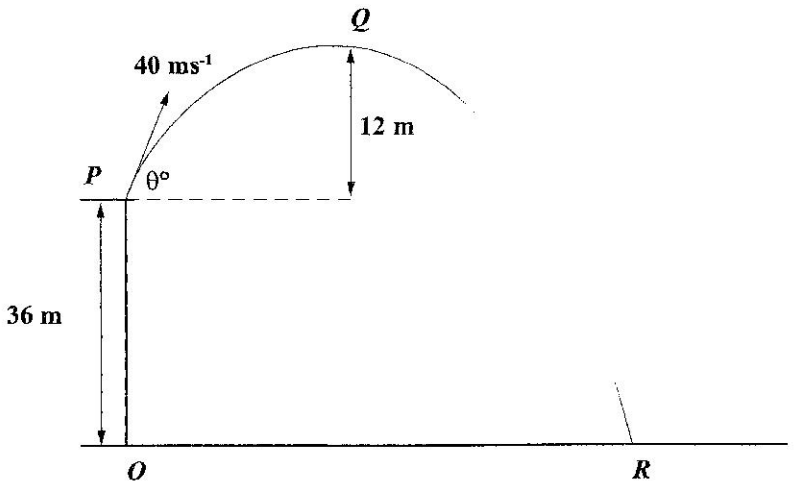
MODULE 3: PARTICLE MECHANICS

Answer BOTH questions.

(Wherever necessary, take $g = 10\ ms^{-2}$.)

5. (a) A ball is projected with a speed of 40 ms^{-1} from a point P on a cliff above horizontal ground. The point O on the ground is vertically below P and OP is 36 m. The ball is projected at an angle $\theta°$ to the horizontal. The point Q is the highest point of the path of the ball and is 12 m above the level of P. The ball moves freely under gravity and hits the ground at the point R, as shown in the following diagram.

Neglecting air resistance, determine



(i) the value of θ

[3 marks]

(ii) the distance *OR*

[5 marks]

(iii) the speed of the ball as it hits the ground at *R*.

[3 marks]

(b) Consider a particle P on the x-axis. The acceleration of P at time t seconds, $t \geq 0$, is $(3t + 5)$ ms^{-2} in the positive x-direction. When $t = 0$, the velocity of P is 2 ms^{-1} in the positive x-direction. When $t = \mathrm{T}$, the velocity of P is 6 ms^{-1} in the positive x-direction.

Calculate the value of T.

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[6 marks]

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(c) An object P, of mass 0.6 kg, is released from rest and slides down a line of greatest slope of a rough plane. The plane is inclined at 30° to the horizontal. When P moves 12 m, its speed is 4 ms^{-1}. Given that friction is the only non-gravitational resistive force acting on P, determine

(i) the work done against friction as the speed of P increases from 0 ms^{-1} to 4 ms^{-1}

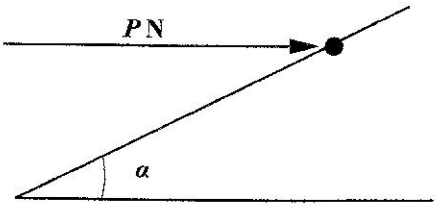
[4 marks]

(ii) the coefficient of friction between the object and the plane.

[4 marks]

Total 25 marks

6. (a) The following diagram shows an object of mass 0.4 kg, held at rest on a fixed rough plane by a horizontal force of magnitude P newtons. The force acts on the vertical plane, containing the line of greatest slope of the inclined plane which passes through the particle. The plane is inclined to the horizontal at an angle α, where $\tan \alpha = \frac{3}{4}$.



The coefficient of friction between the particle and the plane is $\frac{1}{3}$.

Given that the particle is on the point of sliding up the plane, find the magnitude of the normal reaction between the particle and the plane, and the value of P.

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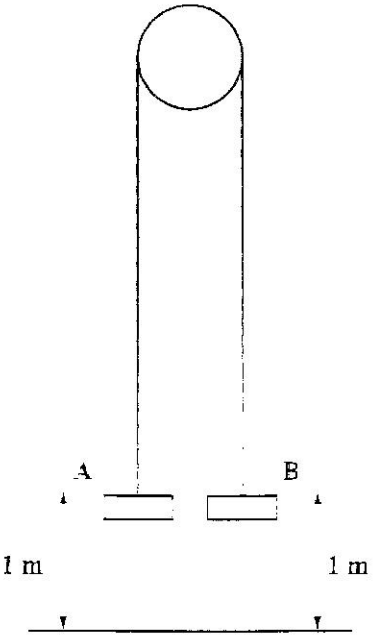
(b) Two particles, A and B have masses 0.4 kg and 0.3 kg respectively. The particles are attached to the ends of a light inextensible string. The string passes over a small smooth pulley which is fixed above a horizontal floor. Both particles are held, with the string taut, at a height of 1 m above the floor, as shown in the following diagram. The particles are released from rest and in the subsequent motion B does not reach the pulley.



(i) Calculate the acceleration of A immediately after the particles are released from rest.

[4 marks]

Find the tension in the string immediately after the particles are released.

[2 marks]

(iii) When the particles have been moving for 0.5 s, the string breaks.

Calculate the greatest height above the floor at which B comes to momentary rest.

[9 marks]

Total 25 marks

END OF TEST

IF YOU FINISH BEFORE TIME IS CALLED, CHECK YOUR WORK ON THIS TEST.